BRAKING OF CONDUCTIVE CLUSTERS MOVING IN CHANNELS IN A NONUNIFORM MAGNETIC FIELD

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The motion of plasma clusters in magnetic fields has long been under intensive study. A wide range of studies on the subject has been carried out in connection with the problems of plasma injection and confinement in a magnetic field (e.g. see [1 and 2]). In these applications the plasma conductivity is large, so that the magnetic Reynolds numbers R_m considerably exceed unity; the rate of diffusion of the magnetic field is sufficiently small.

Many studies of unsteady motions of plasma clusters have been related to the design of propulsion devices (e.g. see [3 ÷ 7]). These studies have been concerned with the conversion of the electrical energy stored in the discharge circuit into kinetic energy of the cluster. The theoretical models employed usually presupposed that the current flowing through the cluster is either uniformly distributed or breaks down into separate pinches. It was assumed in most cases, however, that the flow of current through the cluster is rectilinear.

Interest has recently been growing in another aspect of the problem of motion of plasma clusters, namely, in the flow of conducting gases in shock and electric discharge tubes and in channels with generation of electrical energy in the impulsive discharge process. These motions are characterized by the presence of traveling fronts which interact with the field. This makes it important to investigate the diffusion of the magnetic field into the cluster and its braking as a result of eddy current formation.

Diffusion of a magnetic field into an undeformable cluster moving at constant velocity was investigated in [8]. It was assumed there that the magnetic field was planar and that the moving body was an infinite cylinder of square cross section, so that the electric currents flowed parallel to the cylinder generatrix. The motions of a conducting piston at constant velocity between electrodes were considered in one-dimensional formulation in [9].

In contrast to the above studies, the present paper deals with the braking of clusters in channels whose walls are either everywhere nonconductive or contain two ideally conductive zones (electrodes) connected to an external load. Under such conditions the kinetic energy of the cluster is transformed into Joule heat released inside it and into electrical energy which is fed into the external load. An important factor in this process is the formation of closed electric eddy currents in the cluster as it enters and leaves the magnetic field. The formation of eddy currents explains the braking of the cluster in a channel with nonconductive walls, when the total energy of the cluster (the sum of its kinetic energy and Joule heat) becomes constant.

We note that spatial effects of closed eddy current formation during steady magnetohydrodynamic channel flow have already been investigated (see surveys [10 and 11]). Unsteady electric fields formed during motion of a medium with time-dependent conductivity were investigated in [12]. The gas velocity distribution in these studies was assumed to be known and independent of interaction with the magnetic field. The major emphasis in the present paper is shifted to the problems of dynamics of the medium (its braking) under conditions close to those realized in electric discharge apparatus and devices with impulsive generation of electric energy at moderate gas conductivities. Our model of the medium will be a nondeformable conductive gas cluster which experiences braking in the channel as a result of interaction with the magnetic field.

1. Let us consider the motion of a plasma cluster in a cylindrical channel in the external magnetic field $\mathbf{B}_e$. We assume that variations in the size and shape of the cluster volume are negligible (that the solid-body model is valid) (*). We also assume that the cluster is in contact with the channel walls and that its end surfaces are perpendicular to the channel axis.

The equation of motion of the cluster is of the form

$$M\frac{dV}{dt}=f_\Sigma, \qquad f_\Sigma=\int\limits_{X-l}^{X(t)}\int\limits_F f_x\,dF\,dx \tag{1.1}$$

Here M and l are the mass and length of the cluster; $V(t)$ is its velocity along x; $f_\Sigma(t)$ is the sum axial force acting on the cluster; $f_x(x,t)$ is the volume density of the forces; $X(t)$ is the position of the front end surface; $F=\mathrm{const}$ is the transverse cross-sectional area of the channel.

We assume that the force of friction of the cluster against the channel walls and the resistance of the external medium are much smaller than the electromagnetic forces. In this case

$$f_x=\frac{1}{c}(\mathbf{j}\times\mathbf{B})_x \tag{1.2}$$

Here $\mathbf{B}$ and $\mathbf{j}$ are the magnetic field and electric current density vectors; c is the velocity of light in vacuum.

Multiplying Eq. (1.1) by V, we obtain

$$MV\frac{dV}{dt}=-A, \qquad A=-\int\limits_{X-l}^{X}\int\limits_F f_xV\,dF\,dx \tag{1.3}$$

The quantity A is the work done by the cluster (per unit time) in overcoming the resistance of the magnetic field.

Integrating (1.3) over time from the instant $t=t_1$ when the cluster moves outside the magnetic field zone with the velocity V_* to the instant $t>t_1$, we obtain

$$\frac{1}{2}MV^2+\int\limits_{t_1}^{t}A(t)\,dt=\frac{1}{2}MV_*^2$$

Thus, the sum of the kinetic energy of the cluster and of the work it does to overcome the resistance of the magnetic field remains constant.

In order to find the cluster velocity $V(t)$ we must know the distributions of $\mathbf{j}$ and $\mathbf{B}$ in the interval $X-l<x<X$ at each instant. These distributions can be found from Maxwell's equations

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*) In the case where the time of interaction of the cluster with the field is of the order l/V (l and V are the length and velocity of the cluster), the solid-body model is applicable if the velocity of propagation of perturbations in the plasma is much smaller than V.

$$\operatorname{rot} \mathbf{B}_i = \frac{4\pi}{c} \mathbf{j} + \frac{\varepsilon}{c} \frac{\partial \mathbf{E}}{\partial t}, \qquad \operatorname{div} \mathbf{B}_i = 0$$
$$\operatorname{rot} \mathbf{E} = -\frac{1}{c} \frac{\partial \mathbf{B}_i}{\partial t}, \qquad \mathbf{j} = \sigma\left(\mathbf{E} + \frac{1}{c} \mathbf{v} \times \mathbf{B}\right) \tag{1.4}$$
$$\operatorname{div} \mathbf{B}_e = 0, \quad \operatorname{rot} \mathbf{B}_e = 0, \quad \mathbf{B} = \mathbf{B}_e(x, y, z) + \mathbf{B}_i(x, y, z, t)$$

In these equations $\mathbf{B}_i$ and $\mathbf{B}_e$ are the induced and applied (steady) magnetic fields; $\mathbf{E}$ is the electric field vector. In writing Eqs. (1.4) we assumed that the dielectric constant of the medium ε = const; the magnetic permeability was assumed to equal unity.

Making use of Ohm's law (the fourth relation in (1.4)), we can rewrite Expression (1.3) as

$$A = -\int\limits_{X-l}^{X} \int\limits_{F} f_x V \, dF \, dx = -\int\limits_{X-l}^{X} \int\limits_{F} \left(\frac{1}{c} \mathbf{j} \times \mathbf{B}\right) \mathbf{v} \, dF \, dx =$$
$$= \int\limits_{X-l}^{X} \int\limits_{F} \left(\frac{\mathbf{j}}{\sigma} - \mathbf{E}\right) \mathbf{j} \, dF \, dx = Q + N \tag{1.5}$$
$$Q = \int\limits_{X-l}^{X} \int\limits_{F} \frac{\mathbf{j}^2}{\sigma} dF \, dx, \qquad N = -\int\limits_{X-l}^{X} \int\limits_{F} \mathbf{j}\mathbf{E} \, dF \, dx$$

The quantities Q and N naturally represent the Joule dissipation (per unit time) in the cluster and the electrical power generated in it, respectively.

On the introduction of the vector potential Ω according to Formula $\mathbf{B}_i = \operatorname{rot} \Omega$, the expression for the power N becomes

$$N = \int\limits_{X-l}^{X} \int\limits_{F} \mathbf{j}\left(\nabla\varphi + \frac{1}{c} \frac{\partial \Omega}{\partial t}\right) dF \, dx =$$
$$= \int\limits_{X-l}^{X} \int\limits_{F} \left(\operatorname{div} \varphi \mathbf{j} - \varphi \operatorname{div} \mathbf{j} + \frac{1}{c} \frac{\partial \Omega}{\partial t} \mathbf{j}\right) dF \, dx = N_c + N_i$$
$$N_c = \int\limits_{\Sigma} \varphi j_n \, d\Sigma, \qquad N_i = \int\limits_{X-l}^{X} \int\limits_{F} \left(\frac{\varepsilon\varphi}{4\pi} \operatorname{div} \frac{\partial \mathbf{E}}{\partial t} + \frac{1}{c} \frac{\partial \Omega}{\partial t} \mathbf{j}\right) dF \, dx \tag{1.6}$$

Here φ is the electric potential. In obtaining these formulas we made use of the first relation in (1.4). The quantities N_i and N_c represent the energies lost by the cluster per unit time through induction (N_i) and as a result of energy removal through the channel walls (N_c).

Let us estimate the order of magnitude of the terms appearing in Formulas (1.4) – – (1.6). It is clear that the ratio of the terms containing and not containing the derivative $\partial \mathbf{E}/\partial t$, is of the order $\varepsilon_1 = \varepsilon / 4\pi\sigma_* t_*$ (the asterisk subscripts here and below denote the characteristic values of the parameters). In the problems under consideration which are characterized by the conditions $\sigma_* \approx 10^{-1} \div 10^2$ mho/cm and $t_* \gtrsim 10^{-9}$sec, the quantity ε_1 is much smaller than unity, so that the bias currents can be left out of the electrodynamic equations.

Let us now estimate the transient term in the third equation of system (1.4). We note that in our case this term is related to two processes. The first is the diffusion of the magnetic field into the conductive body moving through it. We assume that the diffusion time is much smaller than the characteristic transit time l/V_*. If this is the case (this condition is clearly equivalent to the inequality $R_m(l) = 4\pi\sigma_* V_* l/c^2 \ll 1$), we can assume that the effects associated with skin layer formation are negligible.

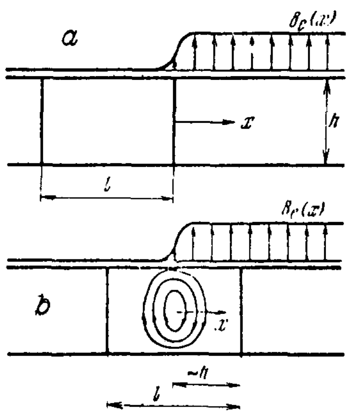

Fig. 1

The second process is the appearance of the induced magnetic field (as a result of displacement of the cluster through the external magnetic field which, by hypothesis, penetrates the cluster "immediately") and of the eddy electric field E_v due to the variation of B_i with respect to t. For example, let us consider the motion of a cluster through a channel with nonconductive walls in the presence of an external magnetic field whose geometry is shown in Fig. 1. In position a (Fig. 1) the field B_i in the cross section $x = 0$ is close to zero. However, when the forward front of the cluster has traveled the distance h (equal in order of magnitude to the transverse dimension of the channel), the eddy electric current which has formed gives rise to the maximum induced magnetic field. The ratio of E_v to the characteristic quantity $V_* B_*/c$ is defined by parameter

$$\varepsilon_2 = R_m(h)\, h/V_* t_*, \quad R_m(h) = 4\pi\sigma_* V_* h/c^2$$

It is clear that the characteristic time of this process is on the order of h/v_* so that $\varepsilon_2 \approx R_m(h)$.

From now on we shall assume that $R_m(h) \ll 1$, so that terms containing the derivatives of $\mathbf{B}$ and Ω with respect to time can be left out of the electrodynamics equations. The electric current vector then becomes potential ($\mathbf{E} = -\nabla\varphi$) and the quantity $\mathbf{B}$ in Ohm's law must be replaced by $\mathbf{B}_e$. The integral characteristics are related by Expression

$$A = Q + N_e \tag{1.7}$$

Formula (1.7) indicates that if energy is not supplied to the cluster from an external source ($N_c \geqslant 0$), then the quantity A is always positive and the cluster is braked by the magnetic field. In passage of a cluster through a channel with nonconductive walls $A = Q$.

The simplified system of electrodynamics equations is a quasi-steadystate system. On application of the operation div to the fourth relation of (1.4) it reduces to Eqs.

$$\Delta\varphi = 0, \qquad \mathbf{j} = \sigma\left(-\nabla\varphi + \frac{1}{c}\mathbf{v}\times\mathbf{B}_e\right)$$
$$j_x = 0 \quad \text{for } x = X - l, \quad x = X \tag{1.8}$$

To these conditions at the end cross sections we must add conditions at the boundary between the cluster and the channel walls.

Eqs. (1.8) coincide with the system of equations which describes the steadystate distribution of an electric field in the channels of magnetohydrodynamic devices and has already been investigated in several papers [10 and 11]. This makes it possible to use available results in solving Eqs. (1.8).

Let us assume that the boundary conditions (which can be written either in differential or in integral form, depending on the type of problem) contain just one new quantity (i.e. one which does not appear in system (1.8)). This is the external resistance R. According to (1.8), the integral electric characteristics A, Q and N_c for a fixed magnetic field geometry are in this case functions of the quantities σ, $^1/cVB_*$, h, X, l, $2a$, R (h and $2a$ are the characteristic dimensions of the channels in the y- and z-directions). Hence, on the basis of similarity and dimensionality theory [13] we have

$$A = \frac{2a\sigma V^2 B_*^2 h^2}{c^2} S\left(\eta, \frac{l}{h}, \frac{2a}{h}, 2aR\sigma\right) \qquad \left(\eta = \frac{X}{h}\right) \tag{1.9}$$

Substituting this expression into (1.3) and noting that $dV/dt = VdV/dX$, and converting to dimensionless variables, we obtain

$$\frac{du}{d\eta} = -\delta S\left(\eta, \frac{l}{h}, \frac{2a}{h}, 2aR\sigma\right) \left(u = \frac{V}{V_*}, \ \eta = \frac{X(t)}{h}, \ \delta = \frac{\sigma B_*^2 h^3 2a}{c^2 M V_*}\right) \tag{1.10}$$

The quantity δ entering into this equation is the magnetohydrodynamic interaction parameter; η is the dimensionless coordinate of the front end surface of the cluster.

Integrating Eq. (1.10) under the boundary condition $u(-\infty) = 1$ according to which the cluster does not interact with the magnetic field at left infinity, we obtain

$$u = 1 - \delta T(\eta), \qquad u^* = \frac{1-u}{\delta}, \qquad T(\eta) = \int_{-\infty}^{\eta} S(\eta)\, d\eta \tag{1.11}$$

If the cluster moves through a channel with nonconductive walls ($A = Q$), then the quantity S is the dimensionless Joule dissipation in the cluster occupying a position $X - l < x < X$. The cluster is braked only as a result of Joule heat release.

If electrodes connected to an external load R are installed at certain positions in the channel walls, braking of the cluster occurs due both to Joule heat release and to the release of electrical energy in the external circuit.

2. Let us suppose that the cluster moves in a channel $-\infty < x < \infty$, $0 < y < h = \text{const}$, $|z| < 2a = \text{const}$ with nonconductive walls in the presence of an external magnetic field $\mathbf{B}_e = (B_x(x, z), 0, B_z(x, z))$. One way to produce such a field by means of an electromagnet whose size in the y-direction exceeds the height h of the channel.

On averaging system (1.8) over the z-coordinate we obtain two-dimensional equations in the average currents and potential [10 and 11] (for simplicity, we use the old letter symbols to represent the average quantities), namely

$$\frac{\partial^2\varphi}{\partial x^2} + \frac{\partial^2\varphi}{\partial y^2} = 0, \qquad j_x = -\sigma\frac{\partial\varphi}{\partial x}, \qquad j_y = -\sigma\frac{\partial\varphi}{\partial y} - \frac{\sigma}{c} VB \tag{2.1}$$

$$j_x = 0 \quad \text{for } x = X, \quad x = X - l \qquad (0 < y < h) \tag{2.2}$$

$$j_y = 0 \quad \text{for } y = 0, \quad y = h \qquad (X - l < x < X)$$

In this system the quantity $B = B(x)$ represents the field component $B_z(x, z)$ averaged over z.

Since $\operatorname{div} \mathbf{j} = 0$, we can introduce the dimensionless "stream function" Φ defined by the relations

$$j_x = \frac{cB_*}{4\pi h} \frac{\partial\Phi}{\partial(y/h)}, \qquad j_y = -\frac{cB_*}{4\pi h} \frac{\partial\Phi}{\partial(x/h)} \tag{2.3}$$

The quantity $B_*\Phi$ is the induced magnetic field.

Let us convert to the dimensionless variables

$$x^\circ = \frac{x}{h}, \quad y^\circ = \frac{y}{h}, \quad \mathbf{j} = \frac{\sigma}{c} VB_* \mathbf{j}^\circ, \quad \varphi = \frac{h}{c} VB_* \varphi^\circ$$
$$B(x) = f(x^\circ) B_*, \qquad X = \eta h, \qquad l = hb \tag{2.4}$$

Omitting the zero superscripts next to the dimensionless quantities for simplicity, we obtain from (2.1),(2.2) the following boundary value problem for the function Φ:

$$\triangle \Phi = R_m \frac{df}{dx} \qquad \left(R_m = \frac{4\pi}{c^2} \sigma V h\right) \tag{2.5}$$

$\Phi = 0$ for $y = 0$, $y = 1$ $(\eta - b < x < \eta)$; $x = \eta - b$, $x = \eta$ $(0 < y < 1)$

The general solution of the system (2.5) can be constructed in the form of trigonometric series. The final result is of the form

$$\Phi(x, y) = R_m \sum_{\nu=1}^{\infty} \Phi_\nu(x) \sin 2r_\nu y, \qquad r_\nu = \frac{\pi}{2}(2\nu - 1) \tag{2.6}$$

$$\Phi_\nu(x) = A_\nu \operatorname{ch} 2r_\nu x + B_\nu \operatorname{sh} 2r_\nu x + \frac{1}{2r_\nu^2} \left\{\exp(2r_\nu x) \int_\eta^x \frac{df}{dx} \exp(-2r_\nu x)\, dx - \right.$$
$$\left. - \exp(-2r_\nu x) \int_{\eta-b}^x \frac{df}{dx} \exp(2r_\nu x)\, dx \right\}$$

$$A_\nu = \frac{1}{\operatorname{sh} 2r_\nu b} [\alpha_\nu \operatorname{sh}(2r_\nu \eta) + \beta_\nu \operatorname{sh}[2r_\nu(b - \eta)]],$$

$$B_\nu = \frac{1}{\operatorname{sh} 2r_\nu b} [\beta_\nu \operatorname{ch}[2r_\nu(b - \eta)] - \alpha_\nu \operatorname{sh}(2r_\nu \eta)]$$

$$\alpha_\nu = \frac{\exp[-2r_\nu(b - \eta)]}{2r_\nu^2} \int_{\eta-b}^{\eta} \frac{df}{dx} \exp(-2r_\nu x)\, dx, \tag{2.7}$$

$$\beta_\nu = \frac{\exp(-2r_\nu \eta)}{2r_\nu^2} \int_{\eta-b}^{\eta} \frac{df}{dx} \exp(2r_\nu x)\, dx$$

The function $S(\eta)$ is given by Expression

$$S(\eta) = -\sum_{\nu=1}^{\infty} \frac{1}{r_\nu} \int_{\eta-b}^{\eta} \Phi_\nu \frac{df}{dx}\, dx \tag{2.8}$$

Let us consider two configurations of the magnetic field,

$$f(x) = H(x) \tag{2.9}$$

$$f(x) = H(x) - H(x - \gamma) \qquad H(x) = \begin{cases} 0 & \text{for } x < 0 \\ 1 & \text{for } x > 0 \end{cases} \tag{2.10}$$

Here function $H(x)$ is a Heaviside unit function, and the quantity γ is the dimensionless length of a segment of the uniform magnetic field.

Relations (2.9) and (2.10) are diagrammed in Figs. 2 and 3.

The derivatives of the function f and the quantity S in these cases are given by Formulas

$$f'(x) = \delta(x), \qquad S(\eta) = -\sum_{\nu=1}^{\infty} \frac{\Phi_\nu(0)}{r_\nu} \tag{2.11}$$

$$f'(x) = \delta(x) - \delta(x-\gamma), \qquad S(\eta) = -\sum_{\nu=1}^{\infty} \frac{1}{r_\nu}[\Phi_\nu(0) - \Phi_\nu(\gamma)] \quad (2.12)$$

Here $\delta(x)$ is Dirac's delta.

Formulas (2.11) correspond to relation (2.9), and (2.12) correspond to realtion (2.10).

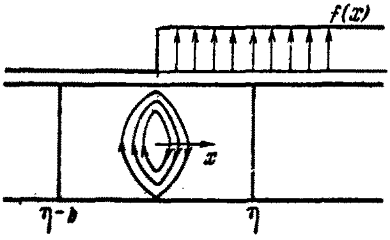

Fig. 2

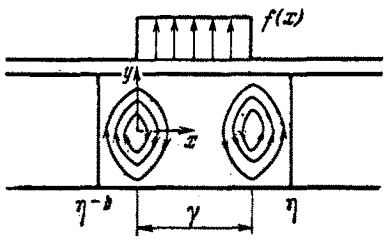

Fig. 3

Let us consider the functions in the case of magnetic field (2.9). If $\eta < 0$ or $\eta > b$ (the cluster is either outside the magnetic field, or lies entirely within it), then the Joule dissipation is equal to zero ($S = 0$). When $\eta < 0$ the electric field in the cluster is equal to zero. If $\eta > b$ and the cluster is enveloped by the uniform field, the electric charge in it experiences separation, and $\mathbf{j} \equiv 0$. The Joule dissipation differs from zero only when the cluster intersects the cross section $x = 0$. The quantity S is in this case given by Formula

$$S = S_1 = \sum_{\nu=1}^{\infty} \frac{\operatorname{sh}(2r_\nu\eta)\operatorname{sh}[2r_\nu(b-\eta)]}{r_\nu^3 \operatorname{sh} 2r_\nu b} \qquad (0 \leqslant \eta \leqslant b) \qquad (2.13)$$

The function $S(\eta)$ increases from $S(0) = 0$ to $S = S_{max}$ for $\eta = {}^1/_2\, b$ and then decreases to zero. The functions $S_1(\eta)$ for several b appear in Fig. 4. The Joule dissipation increases with increasing cluster length.

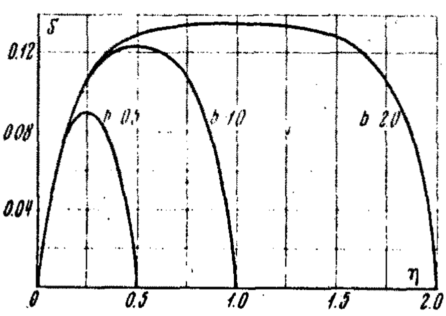

Fig. 4

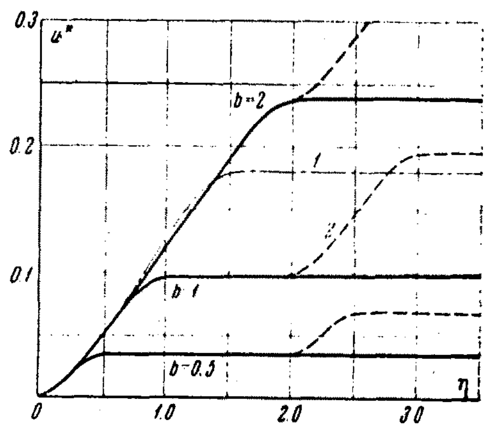

Fig. 5

The function $T(\eta)$ defined by Formula (1.11) is of the form

$$T = 0 \quad \text{for} \quad \eta < 0 \qquad (2.14)$$

$$T = T_1(\eta) = \sum_{\nu=1}^{\infty} \frac{2r_\nu\eta \operatorname{ch}(2r_\nu b) - \operatorname{sh}(2r_\nu\eta)\operatorname{ch}(b-\eta)}{4r_\nu^4 \operatorname{sh}(2r_\nu b)} \quad \text{for} \quad 0 \leqslant \eta \leqslant b$$

$$T = T_1(b) \quad \text{for} \quad \eta \geqslant b$$

Now let us consider the electric field in a cluster in the case of magnetic field (2.10). Here two situations are possible: $\gamma > b$ and $\gamma < b$.

If the magnetic field zone is longer than the cluster ($\gamma > b$), then the current distribution can be found with the aid of the case considered above. In fact, we have

$$S=\begin{cases}0 & (\eta<0)\\ S_1(\eta) & (0<\eta<b)\\ 0 & (b<\eta<\gamma)\\ S_1(\gamma-\eta+b) & (\gamma<\eta<\gamma+b)\\ 0 & (\eta>\gamma+b)\end{cases},\qquad T=\begin{cases}0 & (\eta<0)\\ T_1(\eta) & (0<\eta<b)\\ T_1(b) & (b<\eta<\gamma)\\ 2T_1(b)-T_1(\gamma-\eta+b) & (\gamma<\eta<\gamma+b)\\ 2T_1(b) & (\eta>\gamma+b)\end{cases}\tag{2.15}$$

When the magnetic field zone is smaller than the length of the cluster, it is necessary to have data on Joule dissipation in the position indicated in Fig. 3.

From general solution (2.6), (2.7) and Formula (2.12) we obtain

$$S=S_2(\eta)=2\sum_{\nu=1}^{\infty}\frac{\operatorname{sh} r_\nu\gamma}{r_\nu^3\operatorname{sh}2r_\nu b}\left[\operatorname{sh}[r_\nu(2b-\gamma)]-\operatorname{sh}(r_\nu\gamma)\operatorname{ch}[2r_\nu(b+\gamma-2\eta)]\right]$$
$$(\gamma<\eta<b+\gamma)\tag{2.16}$$

$$T_2(\eta)=\int_\gamma^\eta S_2(\eta)\,d\eta=\sum_{\nu=1}^{\infty}\frac{\operatorname{sh} r_\nu\gamma}{r_\nu^4\operatorname{sh}(2r_\nu b)}\{(\eta-\gamma)\,2r_\nu\operatorname{sh}[r_\nu(2b-\gamma)]-$$
$$-\operatorname{sh}(r_\nu\gamma)\operatorname{sh}[2r_\nu(\eta-\gamma)]\operatorname{ch}[2r_\nu(b-\eta)]\}\tag{2.17}$$

$$T(\eta)=\begin{cases}0 & (\eta<0)\\ T_1(\eta) & (0<\eta<\gamma)\\ T_1(\gamma)+T_2(\eta) & (\gamma<\eta<b)\\ 2T_1(\gamma)+T_2(b)-T_1(\gamma-\eta+b) & (b<\eta<b+\gamma)\\ 2T_1(\gamma)+T_2(b) & (\eta>b+\gamma)\end{cases}\tag{2.18}$$

Fig. 5 shows the relation $u^*=\delta^{-1}(1-u)$, computed from Formulas (2.14) for magnetic field (2.9) for several cluster lengths b (solid curves). All of the curves practically coincide for small η. Thus, when $b\sim 1$ braking of the cluster in the first stage (as it enters the magnetic field zone) is weakly dependent on its length. Subsequent braking of longer clusters is more intense and terminates later (for larger values of η) than is the case with short clusters.

Variation of the cluster velocity in magnetic field (2.10) under the condition $b<\gamma$ is shown in Fig. 5 (broken curves). The computations were carried out using Formula (2.15) for $\gamma=2$. The solid and broken curves coincide in the range $0<\eta<2$. Braking increases with increasing cluster length. For $b=2$ the cluster in the $0<\eta<4$ is braked continuously. If $b<2$, then the cluster travels with constant velocity over a certain portion of its path.

Fig. 5 also shows $u^*(\eta)$ for magnetic field (2.10) computed for $\gamma=0.5$, $b=1$ (dot-and-dash curve 1). Since the cluster is longer than the magnetic field zone, its braking is continuous, and two eddy current zones (at entry into and emergence from the field) arise in the cluster for $0.5<\eta<1$.

Let us compare curves 1 and 2 (curve 2 corresponds to the motion of a cluster of length $b=1$ in a magnetic field of length $\gamma=2$). For $\eta<0.5$ these curves coincide. Then, for $0.5<\eta<1$, curve 1 rises above curve 2 due to the formation of a second eddy zone in the cluster for $\gamma=0.5$. When η lies in the range (1, 1.5), the curves diverge even more, since, if $\gamma=2$, the whole cluster enters the zone of the uniform magnetic field and travels at constant velocity. However, if $\gamma=0.5$, then the cluster leaves the magnetic field zone completely for $\eta>1.5$, while for $\gamma=2$ and $2<\eta<3$ the cluster once again begins to experience braking by the magnetic field. Thus, the sum decrease in velocity turns out to be larger in the case of a long magnetic field.

3. Let us consider the motion of a cluster in a channel of rectangular cross section $\infty<x<\infty$, $0<y<h, |z|<2a$ with walls which are nonconductive except for the areas $0<x<\gamma h$, $y=0$, $|z|<2a$ and $0<x<\gamma h$, $y=h$, $|z|<2a$, where there are electrodes connected to the load R. We assume that the external magnetic field has the same structure as in Section 2, and that upon averaging over the z-coordinate the dimensionless field component $B_z(x, z)$ is given by Formula (2.10). The case under consideration is therefore that when the electrode zone coincides with the magnetic field zone (Fig. 6).

The distribution of the parameters in the plane x, y averaged over z is described by the following system of equations:

$$\frac{\partial^2\varphi}{\partial x^2}+\frac{\partial^2\varphi}{\partial y^2}=0, \quad j_x=-\sigma\frac{\partial\varphi}{\partial x}, \quad j_y=-\sigma\frac{\partial\varphi}{\partial y}-\frac{\sigma}{c}VB_*f \tag{3.1}$$
$$\frac{\partial\varphi}{\partial x}=0 \quad \text{for } x=X, \quad x=X-l$$

$$\frac{\partial\varphi}{\partial y}=0 \quad \text{on } AB,\ NM; \qquad \frac{\partial\varphi}{\partial x}=0 \quad \text{on } BC,\ MD$$
$$\varphi_{BC}-\varphi_{MD}=JR \qquad \left(J=-2a\int\limits_{BC} j_y(x,0)\,dx\right) \tag{3.2}$$

The quantities φ_{BC} and φ_{MD} in relations (3.2) are the potentials of the lower and upper electrodes.

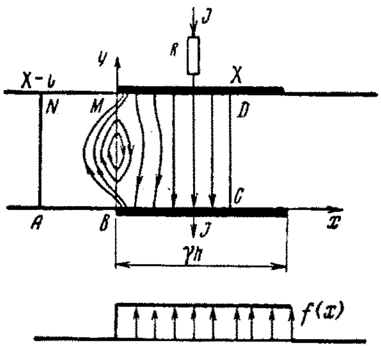

Fig. 6

In addition to the situation depicted in Fig. 6 it is possible to have one in which the cluster bounds only the electrode portions of the walls (for $l<\gamma h$ and $l<X<\gamma h$). In this case the current distribution in the cluster is uniform.

In the same way we can formulate the boundary value problem of the electric field distribution in a cluster as it emerges from the electrode zone (which coincides with the magnetic field zone).

In [10 – 14) it is shown that boundary value problem (3.1) is reducible to a simpler problem which consists in determining the effective internal resistance R_i of the segment $ABCDMN$ on passage of an electric current through the electrodes BC and MD in the absence of a magnetic field. The quantity R_i is given by

$$R_i=(2a\sigma\Phi)^{-1} \tag{3.3}$$

where Φ is a function which depends on the geometric parameters q and $b=l/h$ (q is the dimensionless of the portion of the electrode bounding the cluster). On entry of the cluster into the field zone we have $q=\eta$. For the values $l<\gamma h$ and $l<X<\gamma h$ we have $q=b=\text{const}$.

Using the methods developed in [14], we can show that the electric current J flowing through the cluster is given by

$$J=(2a/c)\,\sigma qVB_* - 2a\sigma U\Phi, \quad U=\varphi_{BC}-\varphi_{MD} \qquad (3.4)$$

Under condition (3.2) we have

$$K=\frac{Uc}{VB_*h}=\frac{qr}{1+r\Phi}, \qquad r=2aR\sigma \qquad (3.5)$$

The function K depends on the argument q which varies with displacement of the cluster, and on the constant parameters r and h. The parameter r characterizes the external load.

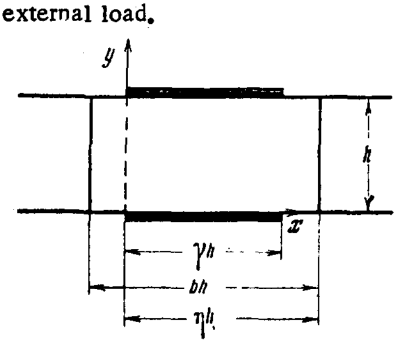

Fig. 7

The quantity A defined by Formula (1.7) can be expressed as

$$A=\frac{2a}{c^2}\sigma B_*^2V^2h^2q\,(1-K) \qquad (3.6)$$

Thus, the function S appearing in relations (1.9)–(1.11) is of the form

$$S=\frac{q\,[1+r\,(\Phi-q)]}{1+r\Phi} \qquad (3.7)$$

Let us cite formulas for determining the quantity Φ. When the cluster is in the position indicated in Fig. 6, we can use the results of [15] to obtain

$$\Phi=\frac{1}{2}\frac{K'(\mu)}{K(\mu)}, \quad \mu=\left(\frac{1-k^2}{1-p^2}\right)^{1/2}, \quad \frac{1}{2b}=\frac{K'(k)}{K(k)} \qquad (3.8)$$

$$F(\alpha,k)=\frac{\eta}{b}K(k) \qquad \left(\alpha=\arccos\left[\frac{p}{k}\left(\frac{1-k^2}{1-p^2}\right)\right]^{1/2}\right)$$

$$F(\theta,k)=\int_0^{\theta}\frac{d\theta}{\sqrt{1-k^2\sin^2\theta}}, \quad K(k)=F\left(\frac{\pi}{2},k\right), \quad K'(k)=K\left(\sqrt{1-k^2}\right) \qquad (3.9)$$

In these formulas $F(\theta, k)$ and $K(k)$ are partial and total elliptic integrals of the first kind, respectively.

The third relation in (3.8) is an equation for determining the quantity $k=k(b)$; the fourth relation yields $p=p(b,\eta)$.

If the cluster is situated in the electrode zone, then $\Phi=q$.

As the cluster emerges from the electrode zone ($\eta>\gamma$, $0<\eta-b<\gamma$), the function Φ is given by Formulas (3.8), (3.9) in which η has been replaced by $\gamma+b-\eta$; finally, if the cluster is longer than the electrode zone, it can have the position shown in Fig. 7, for which

$$\Phi=\frac{F(\theta_1,d)+F(\theta_2,d)}{2K(\tau)}, \quad \sin\theta_1=\sqrt{{}^1/_2(1+p)}, \quad \sin\theta_2=\sqrt{{}^1/_2(1+\varkappa)}$$

$$d=\sqrt{\frac{2(\varkappa+p)}{(1+p)(1+\varkappa)}}, \qquad \tau=\sqrt{\frac{(1-p)(1-\varkappa)}{(1+p)(1+\varkappa)}} \qquad (3.10)$$

$$\frac{1}{b}=\frac{K'(k)}{K(k)}, \quad F\left\{\arccos\frac{p}{k}\left(\frac{1-k^2}{1-p^2}\right)^{1/2}, k\right\}=\frac{2}{b}K(k)(b-\eta)$$

$$F\left\{\arccos\frac{\varkappa}{k}\left(\frac{1-k^2}{1-\varkappa^2}\right)^{1/2}, k\right\}=\frac{2}{b}K(k)(\eta-\gamma) \qquad (3.11)$$

Here Eqs. (3.11) enable us to determine (*) the quantities k, p and $\varkappa$ in succession. It is easy to see that $\partial S / \partial r < 0$, i.e. that the quantity S increases monotonically from $S = q\,[1 - (q/\Phi)]$ to $S = q$ as the external resistance decreases from $R = \infty$ to $R = 0$.

Hence, maximum braking of the cluster occurs with short circuiting ($R = 0$), and minimal braking when $R = \infty$ (in open-circuit operation). However, even in open-circuit operation the quantity S is larger than it is with the cluster in the same position in a channel with nonconductive walls.

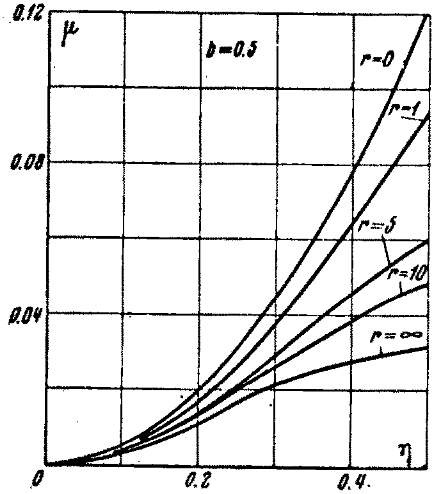

Fig. 8

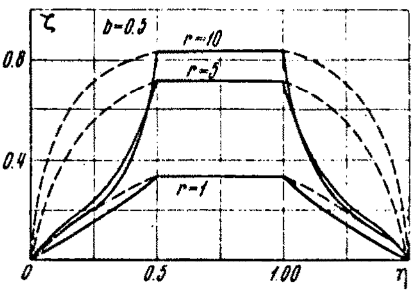

Fig. 9

This is due to the fact that the presence of conductive areas in the walls facilitates the flow of circulating currents in the end zones of the magnetic field.

Let us consider the case where the cluster is shorter than the magnetic field zone ($b < \gamma$). The function $u^* = T\,(\eta)$ defined in (1.11) is of the form

$$u^* = \frac{1}{2}\eta^2 - r\int_0^\eta \frac{\eta^2 d\eta}{1 + r\Phi} = \mu\,(\eta) \qquad (0 \leqslant \eta \leqslant b)$$

$$u^* = \mu\,(b) + \frac{b}{1 + rb}(\eta - b) \qquad (b \leqslant \eta \leqslant \gamma) \tag{3.12}$$

$$u^* = 2\mu\,(b) + \frac{b}{1 + rb}(\gamma - b) - \mu\,(\gamma + b - \eta) \qquad (\gamma \leqslant \eta \leqslant \gamma + b)$$

The functions $\mu\,(\eta)$ characterizing the braking of the cluster on its entry into the electrode zone appear in Fig. 8. Then, on entering the magnetic field zone, the cluster moves at constant velocity if $r = \infty$; its velocity decreases linearly with increasing η, if $r < \infty$. Braking of the cluster on emergence from the electrode zone (as we see from (3.12)) is also described by the function $\mu\,(\eta)$.

On passage of the cluster through the electrode zone, the electric power N given by Formula

$$N = \frac{2a}{c^2}\sigma V^2 B_*{}^2 h^2 q^2 \frac{r}{(1 + r\Phi)^2} \tag{3.13}$$

appears across the external load.

The sum electrical energy W supplied to the external load during the time of passage

*) Formulas (3.10) and (3.11) have been derived independently by I. M. Rutkevich and E. K. Kholshchevnikova.

of the cluster through the magnetic field, and the energy conversion efficiency ξ defined as the ratio N/A are given by Formulas

$$W = \frac{2a}{c^2}\sigma V_*^2 B_*^2 h^3 r \int_{-\infty}^{+\infty} \frac{u(\eta)\, q^2(\eta)\, d\eta}{(1+r\Phi)^2}, \qquad \xi = \frac{rq}{(1+r\Phi)\,[1+r(\Phi-q)]}$$

The functions $\xi(\eta)$ corresponding to the conditions $b = 1$, $\gamma = 1$ appear in Fig. 9. The broken curves denote the electrical energy generating efficiency without allowance for eddy currents.

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